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The study of non-maximally entangled mixed states(NMEMS) in atom-photon interactions

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Abstract

The arena of atom-photon interactions is a vast and potentially useful physical domain for implementing quantum information protocols. Entanglement has been widely observed in quantum optical systems such as cavity quantum electrodynamics. The study of several facets of quantum entanglement generated in atom-photon interactions is very important with the viewpoint of obtaining interesting and useful applications in real physical processes and devices. In this paper we investigate the non-maximally entangled mixed states and the entanglement properties of a pair of two-level atoms going through a cavity environment one after another. The initial joint state of two successive atoms that enter the cavity is separable. Interactions mediated by the cavity photon field environment result in the final two-atom state being of a non-maximally mixed entangled type which are known as entangled mixed states (NMEMS). We consider the Fock state field and thermal field inside the cavity, and calculate the entanglement of formation E_F, the well known measure appropriate for mixed states, of the joint two-atom state as a function of the Rabi-angle gt. The utilities of such generated NMEMS in quantum information processing tasks can be very important and studied further.

Keywords:

Entanglement;
Jaynes CummingsModel;
Rabi-angle (gt);
Non-maximallyentangled mixed states;
Quantum
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1. Introduction

Entanglement plays an pivotal role in quantum mechanics. A pair of particles is said to be entangled in quantum mechanics if its state cannot be expressed as a product of the states of its individual constituents. This was first noted by Einstein, Podolsky and Rosen in 1935 [1]. The preparation and manipulation of these entangled states that have nonclassical and nonlocal properties leads to a better understanding of basic quantum phenomena. For example, complex entangled states, such as the Greenberger, Horne, and Zeilinger triplets of particles [2, 3] are used for tests of quantum nonlocality [4]. Beyond these fundamental aspects, entanglement has become a fundamental resource in quantum information processing [3, 5] and there has been rapid development of this subject in recent years [6].

Entanglement has been widely observed within the framework of quantum optical systems such as cavity quantum electrodynamics. Many beautiful experiments have been carried out, and in recent years, entangled states have been created and verified. Practical realization of various features of quantum entanglement are obtained in atom-photon interactions in optical and microwave cavities [7]. An example that could be highlighted is the generation of a maximally entangled state between two modes in a single cavity using a Rydberg atom coherently interacting with each mode in turn [8]. For practical implementation of quantum information protocols useful in communication and computation[6], entanglement has to be created and preserved between qubits that are well separated, and a recent experimental breakthrough has been obtained by entangling two distant atomic qubits by their interaction with the same photon [9]. From the viewpoint of information processing, quantification of entanglement is an important aspect, and recently some studies have been performed to quantify the entanglement that is obtained in atom-photon interactions in cavities [10]-[17].

In the present paper we will study the dynamical generation of non-maximally entangled mixed states (NMEMS) of two two-level atoms mediated by various cavity fields. Before using these states in quantum information theory it is obvious to study their characteristics. Since the atoms do not interact directly with each other, the properties of the radiation field encountered by them bears crucially on the nature of atomic entanglement. Our main purpose is to focus on the effect of different field dynamics on the magnitude of entanglement of the atom-atom non-maximally entangled mixed states (NMEMS). We take the initial state of the two atoms as separate or product state and compute the entanglement generated between the atoms by the action of the cavity field environment encountered by the atoms while passing through the cavity one after the other. The interaction between the atom and the field is governed by the Jaynes-Cummings model [18] which is experimentally realizable. Note that there is no spatial overlap between the two atoms in this scheme, i.e., the two atoms never interact directly with each other. The generation of nonlocal correlations between the two atoms emerging from the cavity can in general be understood using the Horodecki theorem [6], and the joint two-atom state is known to violate Bell-type inequalities [18]. Since the joint state of the two atoms emanating from the cavity is not a pure state, we quantify the entanglement using the well-known measure appropriate for mixed states, i.e., the entanglement of formation[19]. We investigate how the statistics of different types of radiation fields influence the quantitative dynamics of atomic entanglement.

The structure of the paper is as follows. In Section 2 we review the interaction between two-level atom and single mode radiation field inside a cavity described by the Jaynes-Cummings (JC) model. We briefly discuss a quantifying technique for bipartite entanglement of a mixed quantum state, i.e., the entanglement of formation. In Section2 we describe the model. In Section 3 and 4 we show how the entanglement between two spatially separated atoms is generated by the action of field environment.

We observe robust atom-atom entanglement mediated by the (a) Fock state field, (b) thermal field respectively. We demonstrate how the various field statistics are reflected in two-atom non-maximally entangled mixed states (NMEMS). Several distinctive characteristics of the entanglement generated by the Fock state field and the thermal field are discussed. It is observed that for the cavity low photon number case, the entanglement between the two atoms decreases with increasing average photon number of the field. Finally we conclude our results.

2. Entanglement mediated by the Jaynes-Cummings interaction

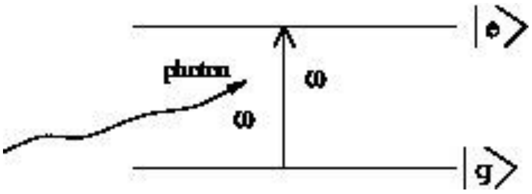


Figure 1: A two-level atom-photon interaction.

The Jaynes-Cummings (JC) model is one of the simplest examples of two interacting quantum systems. It is one of the most studied models in quantum optics because it is an exactly solvable model. Entanglement of the optical field with matter in the JC model has been studied earlier [18].

Here our aim is to study the entanglement between atoms mediated by the optical field, where the light-atom interaction is governed by the JC model. The JC model consists of a two-level atom coupled to a single-mode radiation field inside a cavity. A two level atom is formally analogous to a spin-1/2 system.

The Hamiltonian in the interaction picture reduces to

$H_I = g(\sigma^+ a + \sigma^- a^\dagger)$, where g is the atom-field coupling constant.

A two level atom is formally analogous to a spin-1/ 2 system with two possible states. Let us denote the upper level of the atom as |e> and the lower level as |g>. Here we can write the step up and the step down operator as σ^+ and σ^-, with the commutation relation $[\sigma^+, \sigma^-] = |e\rangle\langle e| - |g\rangle\langle g|$.

Here a and $a^\dagger$ are annihilation and creation operators, respectively, ω is the frequency of the field. The graininess of the radiation field is represented by the photon number state |n> , n = 0, 1, 2, ..., such that $a|n\rangle = \sqrt{n}|n-1\rangle$ and $a^\dagger|n\rangle = \sqrt{(n+1)}|n+1\rangle$. We have considered the quality factor of the cavity Q = ∞ since the cavity-QED related experiments are carried out with cavities with very high Q [7].

3. The Model

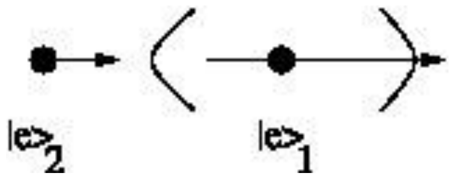


Figure 2: Two atoms prepared in excited states are pass through a single mode cavity one after the other.

We consider a micromaser system in which atoms are sent into the cavity at such a rate that the probability of two atoms being present there is negligibly small. Our purpose here is to show the influence of the photon statistics of the driving fields (radiation field with which the atoms interact) on atomic entanglement. For this sake, we consider the cavity to be of a non-leaky type, that is, $Q = \infty$. In fact, the cavity-QED experiments are very close to such situations [7]. In the following, we consider various kinds of radiation fields. First, we consider a Fock state field to show the effect of a photon number state $|n>$, on atomic entanglement. Next, we consider a thermal field [16, 20].

4. Fock state field

A Fock state is written as $|n>$ with n an integer value, signifying that there are n quanta of excitation in the mode. $|0>$ corresponds to the ground state (no excitation). Let us first consider the passage of the first atom, initially in the excited state $|e>$, through the cavity. The joint atom-field state $|e1> \otimes |n>$. The atom-field wave function evolves under the interaction Hamiltonian

$H_I = g(\sigma^+ a + \sigma^- a^\dagger)$ as

$|\Psi(t)>_{a-f} = \cos(\sqrt{(n+1)}gt\, |e_1, n>$

$+ \sin(\sqrt{(n+1)}gt\, |g_1, n+1>.$

The next atom which enters the cavity interacts with this “changed" field and thus a correlation develops between the two atoms via the cavity field. The joint state of the two atoms and the field is given by

$|\Psi(t)>_{a-a-f} = x_1|e_1, e_2, n> + x_2|e_1, g_2, n+1> + x_3|g1, e2, n+1> + x_4|g1, g2, n+2>,$

where

$x_1 = \cos(\sqrt{n}+1)gt.\ \cos(\sqrt{n}+1)gt,$

$x_2 = \cos(\sqrt{n}+1)gt.\ \sin(\sqrt{n}+1)gt,$

$x_3 = \cos(\sqrt{n}+2)gt.\ \sin(\sqrt{n}+1)gt,$

$x_4 = \sin(\sqrt{n}+1)gt.\ \sin(\sqrt{n}+2)gt.$

The reduced mixed density state of two atoms $\rho(t)_{a-a} = |\Psi(t)_{a-a}><_{a-a}\Psi(t)|$ after tracing over the field is a non-maximally entangled mixed state.

We compute the entanglement of formation E_F for this bipartite two-atom state. In Figure 3, E_f is plotted versus the Rabi angle gt for different values of n.

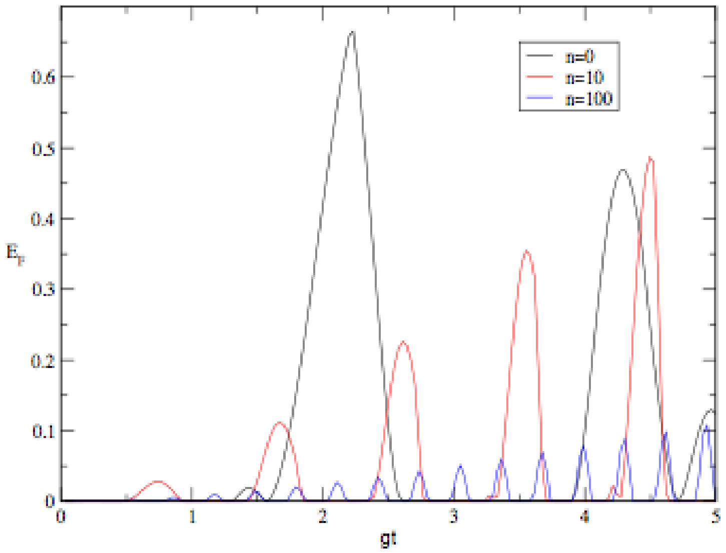


Figure 3 : Atom-atom entanglement $E_F$versus gt. Black line, red line, and blue line indicate EF between two atoms when the cavity Fock states are n = 0, n = 10, and n = 100 respectively.

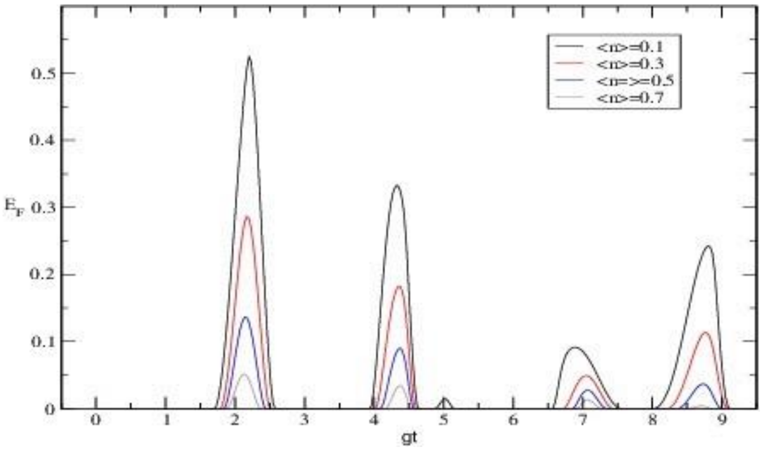
The peaks of the entanglement of formation are reflective of the photon statistics that are typical in micromaser dynamics. We see that entanglement falls off sharply as n increases. The non-classical character of the field for small values of the average photon number n , is reflected in larger entanglement between the two atoms. An interesting comparison can be made with the case of the Tavis–Cummings modewhich is employed when two atoms are present simultaneously inside the cavity. Although simultaneous interaction of two excited atoms with Fock state field never results in two-atom entanglement as was shown by Tessier et al. [17], the notable difference here is that in the JC dynamics modeling the micromaser one always gets two-atom entanglement mediated by the Fock state cavity field, as we see in Fig. 3.

5. Thermal field

The thermal field is the most easily available radiation field, and so, its influence on the entanglement of spins is of much interest. The field at thermal equilibrium obeying Bose-Einstein statistics has an average photon number at temperature T ^{0}K, given by< n >. The photon statistics is governed by the distribution Pn given by $Pn =< n >^n/(1+ < n >)^{n+1}$ [16].

In the same manner we consider a system in which the two atoms are sent into the cavity one after the other. The reduced mixed density state of two atoms after passing through the the thermal cavity field is also a non-maximally entangled mixed state. Here also we compute the entanglement of formation E_F for the above two-atom state and plot it versus the Rabi angle gt for different values of average photon number < n >.

Figure 4 : Atom-atom entanglement of formation E_F mediated by the thermal cavity field is plotted versus gt.



It is interesting to note that the thermal field which has miminimal information can nevertheless entangle qubits that are prepared initially in a separable state. From figure 4 it is clearly seen that E_F In the context of the Tavis-Cummings framework when both the atoms interact simultaneously with the radiation field, Kim et al. [16] have noticed similar trends in the entanglement mediated by the thermal field i.e. $E_F$decreases sharply as <n> increases . Thus both Jaynes-Cummings and the Tavis-Cummings models of atom-photon interaction generate similar entanglement when the radiation field is thermal, whereas for the coherent field case the situation is contrasting as observed in the paper [14].

6. Summary and Discussions

Before concluding, it is instructive to perform a comparative computation of EF mediated by the various cavity radiation fields. As expected, the least magnitude of entanglement is mediated by the thermal field.

To summarize, in this paper we have presented a realistic micromaser-type model where two spatially separated atoms are entangled via a cavity field. The entanglement between the two separate atoms builds up via atom-photon interactions inside the cavity, even though no single atom interacts directly with another. We have computed the two-atom entanglement as measured by the entanglement of formation EF , for the case of four different types of radiation fields, i.e., the Fock state field and the thermal field. Our purpose has been to study the effects of the statistics of the bosonic radiation field on the dynamics of the entanglement of two atomic qubits, i.e., two fermionic systems. Several interesting features of atomic entanglement are observed. We first show that for the Fock state cavity field, entanglement between two successively passing atoms can be generated as a consequence of Jaynes-Cummings (JC) dynamics. This is in contrast to the case when both the atoms reside together inside the cavity when Tavis-Cummings (TC) dynamics for atom-photon interactions is unable to generate atomic entanglement[17]. We then study the entanglement mediated by the thermal radiation field. It is interesting to note that the thermal field which carries minimum information is still able to produce atomic entanglement through both the JC interaction as seen here, and also through the TC interaction as was observed earlier[16]. However, the thermal field having a high value of the average photon number loses its ability to entangle atomic qubits passing through it.

Finally, we would like to reemphasize that the quantitative study of entanglement produced in various types of atom-photon interactions is a relevant arena for investigations. Atom-photon interactions and the generation of entanglement mediated through them are expected to play an important role in possible future practical realizations in the field of quantum communications [9]. The properties of different radiation fields in controlled environments such as that of cavity-QED can be used to manipulate the interactions with atomic qubits [21] and hence control the entanglement produced. Recently, the possibility of entanglement of a thermal radiation field with high temperature phonons associated with moving mirrors of a cavity has been shown [22], brightening the prospects for creating macroscopic entanglement. Even from a purely pedagogical perspective, investigations of such NMEMS in atom-photon interactions producing such NMEMS may lead to the resources of quantum information theoretic protocols like quantum teleportation and quantum dense coding [23, 24]. The utilities of such generated NMEMS in quantum information processing tasks can therefore be studied further.

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